

Code No: 133BD

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, October - 2020

MATHEMATICS - IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, ETM, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 2 hours

Max. Marks: 75

Answer any five questions

All questions carry equal marks

- 1.a) State the necessary conditions for a function $f(z)$ to be analytic. Show that the function $f(z) = \overline{|xy|}$ is not analytic at the origin, although the Cauchy-Riemann equations are satisfied.
- b) Determine the analytic function $w = u + iv$, if $v = \log(x^2 + y^2) + x - 2y$. [8+7]
- 2.a) If $f(z)$ is an analytic function of z , prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \log f'(z) = 0$.
- b) If $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$ and $f(z) = u + iv$ is an analytic function of z , then find $f(z)$ in terms of z . [7+8]
- 3.a) Find the Laurent series that represents the function $f(z) = z^2 \sin \frac{1}{z^2}$ in the domain $0 < z < \infty$.
- b) Evaluate the integral $\int_C \frac{\cos \pi z}{z(z^2+1)} dz$ where C is the circle $|z| = 2$, described in the positive sense. [8+7]
- 4.a) Find the Taylor series expansion of the function $f(z) = \sin^3 z$ about $z = 0$.
- b) Evaluate the integral $\int_C \frac{\cos^2 z}{1+z^2} dz$, where C is the closed circle $|z| = \frac{1}{2}$. [8+7]
5. Evaluate the real integral using contour integration $\int_0^{\infty} \frac{\cos ax}{1+x^2} dx$. [15]
6. Find the bilinear transformation which maps the points $(0, 1, \infty)$ into the points $1, -z, -i$. Find the image of the line $y = 5x$ under this transformation. [15]
- 7.a) Find the Fourier series of $f(x) = \begin{cases} 1+x, & 0 < x < \pi \\ 1-x, & -\pi < x < 0 \end{cases}$.
Deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.
- b) Express the function $f(x) = \begin{cases} 1 & \text{for } x \leq 1, \\ 0 & \text{for } x > 1, \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$. [8+7]
8. Write down the one dimensional heat equation. Find the temperature $u(x, t)$ in a slab whose ends $x = 0$ and $x = L$ are kept at zero temperature and whose initial temperature $f(x)$ is given by [15]
- $$f(x) = \begin{cases} k, & \text{when } 0 < x < \frac{1}{2}L \\ 0, & \text{when } \frac{1}{2}L < x < L \end{cases}$$

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